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Gravity modifications from extra dimensions*

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Abstract
Lowering the string scale in the TeV region provides a theoretical framework for solving the mass hierarchy problem and unifying all interactions. The apparent weakness of gravity can then be accounted for by the existence of large internal dimensions, in the submillimetre region, and transverse to a braneworld where our universe must be confined. I review the main properties of this scenario and its implications for observations at both particle colliders, and in non-accelerator gravity experiments. Such effects are for instance the production of Kaluza–Klein resonances, graviton emission in the bulk of extra dimensions, and a radical change of gravitational forces in the submillimetre range. I also discuss localization of gravity in the presence of infinite size extra dimensions that can modify Newton's law at cosmological distance scales.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

During the last few decades, physics beyond the standard model (SM) was guided from the problem of mass hierarchy. This can be formulated as the question of why gravity appears to us so weak compared to the other three known fundamental interactions corresponding to the electromagnetic, weak and strong nuclear forces. Indeed, gravitational interactions are suppressed by a very high-energy scale, the Planck mass $M_P \sim 10^{19}$ GeV, associated with a length $l_P \sim 10^{-35}$ m, where they are expected to become important. In a quantum theory, the hierarchy implies a severe fine tuning of the fundamental parameters in more than 30 decimal places in order to keep the masses of elementary particles at their observed values. The reason is that quantum radiative corrections to all masses generated by the Higgs vacuum expectation value (VEV) are proportional to the ultraviolet cutoff which in the presence of gravity is fixed

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by the Planck mass. As a result, all masses are 'attracted' to become about 10^{16} times heavier than their observed values.

Besides compositeness, there are two main ideas that have been proposed and studied extensively during the last years, corresponding to different approaches of explaining the mass hierarchy problem. (1) Low-energy supersymmetry with all superparticle masses in the TeV region. Indeed, in the limit of exact supersymmetry, quadratically divergent corrections to the Higgs self-energy are exactly cancelled, while in the softly broken case, they are cut off by the supersymmetry breaking mass splittings. (2) TeV scale strings, in which quadratic divergences are cut off by the string scale and low-energy supersymmetry is not needed. Both ideas are experimentally testable at high-energy particle colliders and in particular at LHC. Below, I discuss their implementation in string theory.

The appropriate and most convenient framework for low-energy supersymmetry and grand unification is the perturbative heterotic string. Indeed, in this theory, gravity and gauge interactions have the same origin, as massless modes of the closed heterotic string, and they are unified at the string scale M_s. As a result, the Planck mass M_P is predicted to be proportional to M_s:

$$M_P = M_s/g, \tag{1}$$

where g is the gauge coupling. In the simplest constructions all gauge couplings are the same at the string scale, given by the four-dimensional (4D) string coupling, and thus no grand unified group is needed for unification. In our conventions $\alpha_{\rm GUT} = g^2 \simeq 0.04$, leading to a discrepancy between the string and grand unification scale $M_{\rm GUT}$ by almost two orders of magnitude. Explaining this gap introduces in general new parameters or a new scale, and the predictive power is essentially lost. This is the main defect of this framework, which remains through an open and interesting possibility.

The other idea can be naturally realized in the framework of type I string theory with D-branes. Unlike in the heterotic string, gauge and gravitational interactions have now different origin. The latter are described again by closed strings, while the former emerge as excitations of open strings with endpoints confined on D-branes [1]. This leads to a braneworld description of our universe, which should be localized on a hypersurface, i.e. a membrane extended in p spatial dimensions, called p-brane (see figure 1). Closed strings propagate in all nine dimensions of string theory: in those extended along the p-brane, called parallel, as well as in the transverse ones. In contrast, open strings are attached with the p-brane. Obviously, our p-braneworld must have at least the three known dimensions of space. But it may contain more: the extra $d_\parallel = p-3$ parallel dimensions must have a finite size, in order to be unobservable at present energies, and can be as large as $\text{TeV}^{-1} \sim 10^{-18}$ m [2]. On the other hand, transverse dimensions interact with us only gravitationally and experimental bounds are much weaker: their size should be less than about 0.1 mm [3]. In the following, I review the main properties and experimental signatures of low string scale models [4, 5].

2. Framework

In type I theory, the different origin of gauge and gravitational interactions implies that the relation between the Planck and string scales is not linear as (1) of the heterotic string. The requirement that string theory should be weakly coupled, constrain the size of all parallel dimensions to be of order of the string length, while transverse dimensions remain unrestricted. Assuming an isotropic transverse space of $n = 9 - p$ compact dimensions of common radius

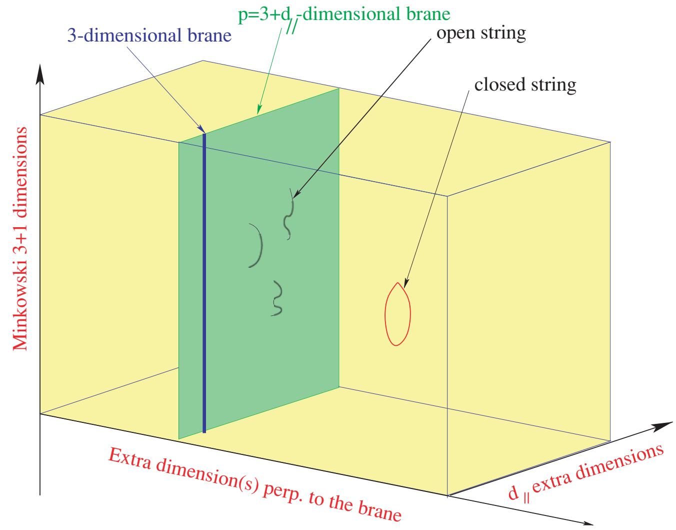

Figure 1. Braneworld.

$R_\perp$, one finds

$$M_P^2 = \frac{1}{g^4} M_s^{2+n} R_\perp^n, \qquad g_s \simeq g^2, \tag{2}$$

where g_s is the string coupling. It follows that the type I string scale can be chosen hierarchically smaller than the Planck mass [6, 4] at the expense of introducing extra large transverse dimensions felt only by gravity, while keeping the string coupling small [4]. The weakness of 4D gravity compared to gauge interactions (ratio M_W/M_P) is then attributed to the largeness of the transverse space $R_\perp$ compared to the string length $l_s = M_s^{-1}$.

An important property of these models is that gravity becomes effectively $(4+n)$-dimensional with a strength comparable to those of gauge interactions at the string scale. The first relation of equation (2) can be understood as a consequence of the $(4+n)$-dimensional Gauss law for gravity, with

$$M_*^{(4+n)} = M_s^{2+n}/g^4 \tag{3}$$

being the effective scale of gravity in $4+n$ dimensions. Taking $M_s \simeq 1$ TeV, one finds a size for the extra dimensions $R_\perp$ varying from 10^8 km, 0.1 mm, down to a Fermi for $n = 1, 2$, or six large dimensions, respectively. This shows that while $n = 1$ is excluded, $n \geqslant 2$ is allowed by present experimental bounds on gravitational forces that tested Newton's law at 130 μm [3, 7]. Thus, in these models, gravity appears to us very weak at macroscopic scales because its intensity is spread in the 'hidden' extra dimensions. At distances shorter than $R_\perp$, it should deviate from Newton's law, which may be possible to explore in laboratory experiments.

3. Experimental implications in accelerators

The main experimental signal is gravitational radiation in the bulk from any physical process on the world-brane. In fact, the very existence of branes breaks translation invariance in the transverse dimensions and gravitons can be emitted from the brane into the bulk. During a collision of centre-of-mass energy $\sqrt{s}$, there are $\sim(\sqrt{s}R_\perp)^n$ KK excitations of gravitons with tiny masses, that can be emitted. Each of these states looks from the 4D point of view as

Table 1. Limits on $R_\perp$ in mm.

Experiment	$n = 2$	$n = 4$	$n = 6$
	Collider bounds		
LEP 2	5×10^{-1}	2×10^{-8}	7×10^{-11}
Tevatron	5×10^{-1}	10^{-8}	4×10^{-11}
LHC	4×10^{-3}	6×10^{-10}	3×10^{-12}
NLC	10^{-2}	10^{-9}	6×10^{-12}
	Present non-collider bounds		
SN1987A	3×10^{-4}	10^{-8}	6×10^{-10}
COMPTEL	5×10^{-5}	–	–

a massive, quasi-stable, extremely weakly coupled (s/M_P^2 suppressed) particle that escapes from the detector. The total effect is a missing-energy cross-section roughly of order:

$$\frac{(\sqrt{s}R_\perp)^n}{M_P^2} \sim \frac{1}{s}\left(\frac{\sqrt{s}}{M_s}\right)^{n+2}. \tag{4}$$

Explicit computation of these effects [8] leads to the bounds given in table 1. However, larger radii are allowed if one relaxes the assumption of isotropy, by taking for instance two large dimensions with different radii.

In table 1, there are also included astrophysical and cosmological bounds. Astrophysical bounds [9, 10] arise from the requirement that the radiation of gravitons should not carry on too much of the gravitational binding energy released during core collapse of supernovae. The best cosmological bound [11] is obtained from requiring that the decay of bulk gravitons to photons does not generate a spike in the energy spectrum of the photon background measured by the COMPTEL instrument.

At energies higher than the string scale, new spectacular phenomena are expected to occur, related to string physics and quantum gravity effects, such as possible micro-black hole production [12], or even signals of gravitational UV fixed points [13]. Particle accelerators would then become the best tools for studying quantum gravity and string theory.

4. Supersymmetry in the bulk and short range forces

Besides the spectacular predictions in accelerators, there are also modifications of gravitation in the submillimetre range, which can be tested in 'table-top' experiments that measure gravity at short distances. There are three categories of such predictions.

(i) Deviations from Newton's law $1/r^2$ behaviour to $1/r^{2+n}$, which can be observable for $n = 2$ large transverse dimensions of submillimetre size.

(ii) New scalar forces in the submillimetre range, related to the mechanism of supersymmetry breaking, and mediated by light scalar fields φ with masses [4, 14]

$$m_\varphi \simeq \frac{m_{\text{susy}}^2}{M_P} \simeq 10^{-4} - 10^{-6}\ \text{eV}, \tag{5}$$

for a supersymmetry breaking scale $m_{\text{susy}} \simeq 1 - 10$ TeV. They correspond to Compton wavelengths of 1 mm to 10 μm. m_{susy} can be either $1/R_\parallel$ if supersymmetry is broken by compactification [14], or the string scale if it is broken 'maximally' on our world-brane [4]. A universal attractive scalar force is mediated by the radion modulus $\varphi \equiv M_P \ln R$, with R being the radius of the longitudinal or transverse dimension(s). In the former case,

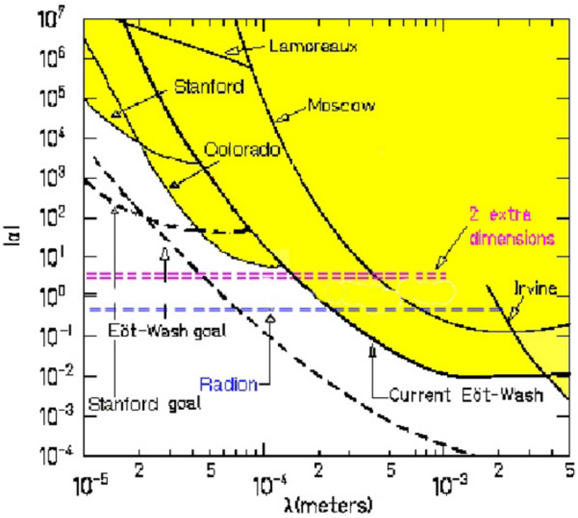

Figure 2. Present limits on non-Newtonian forces at short distances (yellow regions), as a function of their range λ and their strength relative to gravity α. The limits are compared to new forces mediated by the graviton in the case of two large extra dimensions, and by the radion.

result (5) follows from the behaviour of the vacuum energy density $\Lambda \sim 1/R_\parallel^4$ for large $R_\parallel$ (up to logarithmic corrections). In the latter, supersymmetry is broken primarily on the brane, and thus its transmission to the bulk is gravitationally suppressed, leading to (5). For $n = 2$, there may be an enhancement factor of the radion mass by $\ln R_\perp M_s \simeq 30$ decreasing its wavelength by an order of magnitude [15].

The coupling of the radius modulus to matter relative to gravity can be easily computed and is given by

$$\sqrt{\alpha_\varphi} = \frac{1}{M}\frac{\partial M}{\partial \varphi}, \qquad \alpha_\varphi = \begin{cases} \dfrac{\partial \ln \Lambda_{\rm QCD}}{\partial \ln R} \simeq \dfrac{1}{3} & \text{for} \quad R_\parallel \\ \dfrac{2n}{n+2} = 1 - 1.5 & \text{for} \quad R_\perp, \end{cases} \tag{6}$$

where M denotes a generic physical mass. Such a force can be tested in micro-gravity experiments and should be contrasted with the change of Newton's law due the presence of extra dimensions that is observable only for $n = 2$ [3, 7]. The resulting bounds from an analysis of the radion effects are [3]

$$M_* \gtrsim 3 - 4.5 \text{ TeV} \qquad \text{for} \quad n = 2 - 6. \tag{7}$$

In principle, there can be other light moduli which couple with even larger strengths. For example the dilaton, whose VEV determines the string coupling, if it does not acquire large mass from some dynamical supersymmetric mechanism, can lead to a force of strength 2000 times bigger than gravity [16].

(iii) Non-universal repulsive forces much stronger than gravity are mediated by possible Abelian gauge fields in the bulk [9, 17]. Such fields acquire tiny masses of the order of M_s^2/M_P, as in (5), due to brane-localized anomalies [17]. Although their gauge coupling is infinitesimally small, $g_A \sim M_s/M_P \simeq 10^{-16}$, it is still bigger than the gravitational coupling E/M_P for typical energies $E \sim 1$ GeV, and the strength of the new force would be $10^6 - 10^8$ stronger than gravity. This is an interesting region which will be soon explored in micro-gravity experiments (see figure 2).

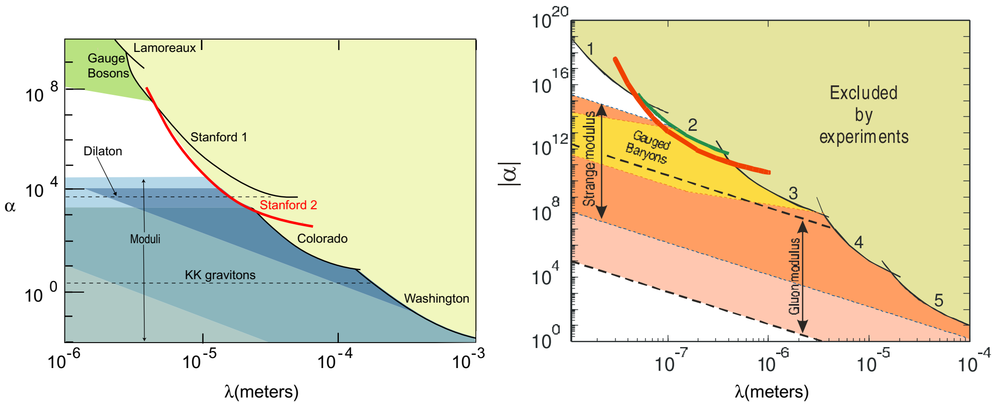

Figure 3. Bounds on non-Newtonian forces in the range 6–20 μm (Smullin *et al* in [7]) and 200 nm (Decca *et al* in [7]). Curves 4 and 5 correspond to Stanford and Colorado experiments, respectively.

In figure 2 we depict the actual information from previous, present and upcoming experiments [7, 15]. The solid lines indicate the present limits from the experiments indicated. The excluded regions lie above these solid lines. Measuring gravitational strength forces at short distances is challenging. The dashed thick lines give the expected sensitivity of the various experiments, which will improve the actual limits by roughly two orders of magnitude, while the horizontal dashed lines correspond to the theoretical predictions for the graviton in the case $n = 2$ and for the radion in the transverse case. These limits are compared to those obtained from particle accelerator experiments in table 1. Finally, in figure 3, we display recent improved bounds for new forces at very short distances by focusing on the right-hand side of figure 2, near the origin [7].

5. Non-compact extra dimensions and localized gravity

There are several motivations to study localization of gravity in non-compact extra dimensions: (i) it avoids the problem of fixing the moduli associated with the size of the compactification manifold; (ii) it provides a new approach to the mass hierarchy problem; (iii) there are modifications of gravity at large distances that may have interesting observational consequences. Two types of models have been studied: warped metrics in curved space [18], and infinite size extra dimensions in flat space [19]. The former, although largely inspired by stringy developments and having used many string-theoretic techniques, have not yet a clear and calculable string theory realization [20]. In any case, since curved space is always difficult to handle in string theory[2], in the following we concentrate mainly on the latter, formulated in flat space with gravity localized on a subspace of the bulk. It turns out that these models of induced gravity have an interesting string theory realization [22] that we describe below.

5.1. The induced gravity model

The DGP model and its generalizations are specified by a bulk Einstein–Hilbert (EH) term and a four-dimensional EH term [19]:

[2] The issue of semiclassical stability of large but curved extra dimensions was studied in [21].

$$M^{2+n}\int_{\mathcal{M}_{4+n}} \mathrm{d}^4x\,\mathrm{d}^n y\sqrt{G}\mathcal{R}_{(4+n)} + M_P^2\int_{\mathcal{M}_4} \mathrm{d}^4x\sqrt{g}\mathcal{R}_{(4)}; \qquad M_P^2 \equiv r_c^n M^{2+n} \tag{8}$$

with M and M_P being the (possibly independent) respective Planck scales. The scale $M \geqslant 1$ TeV would be related to the short-distance scale below which UV quantum gravity or stringy effects are important. The four-dimensional metric is the restriction of the bulk metric $g_{\mu\nu} = G_{\mu\nu}|$ and we assume the WORLD[3] rigid, allowing the gauge $G_{i\mu}| = 0$ with $i \geqslant 5$. Finally, only intrinsic curvature terms are omitted but no Gibbons–Hawking term is needed.

5.1.1. Codimension one. In the case of codimension one bulk ($n = 1$) and δ-function localization, it is easy to see that r_c is a crossover scale where gravity changes behaviour on the WORLD. Indeed, by Fourrier transform the quadratic part of the action (8) with respect to the 4D position x, at the WORLD position $y = 0$, one obtains $M^{2+n}\left(p^{2-n} + r_c^n p^2\right)$, where p is the 4D momentum. It follows that for distances smaller than r_c (large momenta), the first term becomes irrelevant and the graviton propagator on the 'brane' exhibits four-dimensional behaviour ($1/p^2$) with Planck constant $M_P = M^3 r_c$. In contrast, at large distances, the first term becomes dominant and the graviton propagator acquires a five-dimensional fall-off ($1/p$) with Planck constant M. Imposing r_c to be larger than the size of the universe, $r_c \gtrsim 10^{28}$ cm, one finds $M \lesssim 100$ MeV, which seems to be in conflict with experimental bounds. However, there were arguments that these bounds can be evaded, even for values of the fundamental scale $M^{-1} \sim 1$ mm that one may need for suppressing the quantum corrections of the cosmological constant [19].

On the other hand, in the presence of non-zero brane thickness w, a new crossover length scale seems to appear, $R_c \sim (w r_c)^{1/2}$ [23] or $r_c^{3/5} w^{2/5}$ [24]. Below this scale, the theory acquires either again a five-dimensional behaviour, or a strong coupling regime. For $r_c \sim 10^{28}$ cm, the new crossover scale is of order $R_c \sim 10^{-4} - 10$ m.

5.1.2. Higher codimension. The situation changes drastically for more than one non-compact bulk dimensions, $n > 1$, due to the ultraviolet properties of the higher dimensional theories. Indeed, from the action (8), the effective potential between two test masses in four dimensions

$$\int [\mathrm{d}^3x]\,\mathrm{e}^{-\mathrm{i}p\cdot x} V(x) = \frac{D(p)}{1 + r_c^n p^2 D(p)}\left[\tilde{T}_{\mu\nu}T^{\mu\nu} - \frac{1}{2+n}\tilde{T}_\mu^\mu T_\nu^\nu\right]; \qquad D(p) = \int [\mathrm{d}^n q]\frac{f_w(q)}{p^2+q^2} \tag{9}$$

is a function of the bulk graviton retarded Green's function $G(x, 0; 0, 0) = \int [\mathrm{d}^4 p]\,\mathrm{e}^{\mathrm{i}p\cdot x} D(p)$ evaluated for two points localized on the WORLD ($y = y' = 0$). The integral over q in (9) is UV-divergent for $n > 1$ unless there is a non-trivial brane thickness profile $f_w(q)$ of width w. If the four-dimensional WORLD has zero thickness, $f_w(q) \sim 1$, the bulk graviton does not have a normalizable wavefunction. It therefore cannot contribute to the induced potential, which always takes the form $V(p) \sim 1/p^2$ and Newton's law remains four dimensional at all distances.

For a non-zero thickness w, there is only one crossover length scale, R_c:

$$R_c = w\left(\frac{r_c}{w}\right)^{\frac{n}{2}}, \tag{10}$$

above which one obtains a higher dimensional behaviour [25]. Therefore the effective potential presents two regimes: (i) at short distances ($w \ll r \ll R_c$) the gravitational interactions are

[3] We avoid calling $\mathcal{M}_4$ a brane because, as we will see below, gravity localizes on singularities of the internal manifold, such as orbifold fixed points. Branes with localized matter can be introduced independently of gravity localization.

mediated by the localized four-dimensional graviton and Newton's potential on the WORLD is given by $V(r) \sim 1/r$ and, (ii) at large distances ($r \gg R_c$) the modes of the bulk graviton dominate, changing the potential. Note that for $n = 1$ expressions (9) and (9) are finite and unambiguously give $V(r) \sim 1/r$ for $r \gg r_c$. For a codimension greater than 1, the precise behaviour for large-distance interactions depends *crucially* on the UV completion of the theory. At this point we stress a fundamental difference with the *finite extra dimensions* scenarios. In these cases Newton's law gets higher dimensional at distances smaller than the characteristic size of the extra dimensions. This is precisely the opposite of the case of infinite volume extra dimensions that we discuss here.

As mentioned above, for higher codimension, there is an interplay between UV regularization and IR behaviour of the theory. Indeed, several works in the literature raised unitarity [26] and strong-coupling problems [27] which depend crucially on the UV completion of the theory. A unitary UV regularization for the higher codimension version of the model has been proposed in [28]. It would be interesting to address these questions in a precise string theory context. Actually, using for UV cutoff on the 'brane' the 4D Planck length $w \sim l_P$, one gets for the crossover scale (10): $R_c \sim M^{-1}(M_P/M)^{n/2}$. Putting $M \gtrsim 1$ TeV leads to $R_c \lesssim 10^{8(n-2)}$ cm. Imposing $R_c \gtrsim 10^{28}$ cm, one then finds that the number of extra dimensions must be at least six, $n \geqslant 6$, which is realized nicely in string theory and provides an additional motivation for studying possible string theory realizations.

5.2. *String theory realization*

In the following, we explain how to realize the gravity induced model (8) with $n \geqslant 6$ as the low-energy effective action of string theory on a non-compact six-dimensional manifold $\mathcal{M}_6$ [22]. We work in the context of $\mathcal{N} = 2$ supergravities in four dimensions but the mechanism for localizing gravity is independent of the number of supersymmetries. Of course for $\mathcal{N} \geqslant 3$ supersymmetries, there is no localization. We also start with the compact case and take the decompactification limit. The localized properties are then encoded in the different volume dependences.

In string perturbation, corrections to the four-dimensional Planck mass are in general very restrictive. In the heterotic string, they vanish to all orders in perturbation theory [29]; in type I theory, there are moduli-dependent corrections generated by open strings [30], but they vanish when the manifold $\mathcal{M}_6$ is decompactified; in type II theories, they are constant, independent of the moduli of the manifold $\mathcal{M}_6$, and receive contributions only from tree and one-loop levels that we describe below (at least for supersymmetric backgrounds) [22, 31]. Finally, in the context of M-theory, one obtains a similar localized action of gravity kinetic terms in five dimensions, corresponding to the strong-coupling limit of type IIA string [22].

The origin of the two EH terms in (8) can be traced back to the perturbative corrections to the eight-derivative effective action of type II strings in ten dimensions. These corrections include the tree-level and one-loop terms given by

$$\frac{1}{l_s^8}\int_{M_{10}} \frac{1}{g_s^2}\mathcal{R}_{(10)} - \frac{1}{l_s^2}\int_{M_{10}} \left(\frac{2\zeta(3)}{g_s^2} \mp 4\zeta(2)\right) R\wedge R\wedge R\wedge R\wedge \mathrm{e}\wedge \mathrm{e} + \cdots, \tag{11}$$

where ϕ is the dilaton field determining the string coupling $g_s = \mathrm{e}^{\langle\phi\rangle}$, and the $\pm$ sign corresponds to the type IIA/B theory. On a direct product spacetime $\mathcal{M}_6 \times \mathbb{R}^4$, at the level of zero modes, the second term in (11) splits as

$$\int_{M_6} R\wedge R\wedge R \times \int_{M_4} \mathcal{R}_{(4)} = \chi \int_{M_4} \mathcal{R}_{(4)}, \tag{12}$$

where χ is the Euler number of the M_6 compactification manifold. We thus obtain the expressions for the Planck masses M and M_p:

$$M^2 \sim M_s^2/g_s^{1/2}; \qquad M_P^2 \sim \chi\left(\frac{c_0}{g_s^2}+c_1\right)M_s^2, \tag{13}$$

with $c_0 = -2\zeta(3)$ and $c_1 = \pm 4\zeta(2) = \pm 2\pi^2/3$.

It is interesting that the appearance of the induced 4D localized term preserves $\mathcal{N} = 2$ supersymmetry and is independent of the localization mechanism of matter fields (for instance on D-branes). Localization requires the internal space M_6 to have a non-zero Euler characteristic $\chi \neq 0$. Actually, in type IIA/B compactified on a Calabi-Yau manifold, χ counts the difference between the numbers of $\mathcal{N} = 2$ vector multiplets and hypermultiplets: $\chi = \pm 4(n_V - n_H)$ (where the graviton multiplet counts as one vector). Moreover, in the non-compact limit, the Euler number can in general split in different singular points of the internal space, $\chi = \sum_I \chi_I$, giving rise to different localized terms at various points y_I of the internal space. A number of conclusions (confirmed by string calculations in [22]) can be reached by looking closely at (11)–(13):

- ▹ $M_p \gg M$ requires a large non-zero Euler characteristic for M_6, and/or a weak string coupling constant $g_s \to 0$.
- ▹ Since χ is a topological invariant the localized $\mathcal{R}_{(4)}$ term coming from the closed string sector is universal, independent of the background geometry and dependent only on the internal topology. It is a matter of simple inspection to see that if one wants to have a localized EH term in less than ten dimensions, namely something linear in curvature, with non-compact internal space in all directions, *the only possible dimension is four* (or five in the strong-coupling M-theory limit).
- ▹ In order to find the width w of the localized term, one has to do a separate analysis. On general grounds, using dimensional analysis in the limit $M_P \to \infty$, one expects the effective width to vanish as a power of $l_P \equiv M_P^{-1}$: $w \sim l_P^{\nu}/l_s^{\nu-1}$ with $\nu > 0$. The computation of ν for a general Calabi-Yau space, besides its technical difficulty, presents an additional important complication: from the expression (13), $l_P \sim g_s l_s$ in the weak-coupling limit. Thus, w vanishes in perturbation theory and one has to perform a non-perturbative analysis to extract its behaviour. Alternatively, one can examine the case of orbifolds. In this limit, $c_0 = 0$, $l_P \sim l_s$, and the hierarchy $M_P > M$ is achieved only in the limit of large χ. One then finds that the width is given by the four-dimensional induced Planck mass

$$w \simeq l_P = l_s \chi^{-1/2}, \tag{14}$$

and the power $\nu = 1$.

5.2.1. Summary of the results. Using $w \sim l_P$ and the relations (13) in the weak-coupling limit (with $c_0 \neq 0$), the crossover radius of equation (10) is given by the string parameters $(n = 6)$

$$R_c = \frac{r_c^3}{w^2} \sim g_s \frac{l_s^4}{l_P^3} \simeq g_s \times 10^{32}\ \text{cm}, \tag{15}$$

for $M_s \simeq 1$ TeV. Because R_c has to be of cosmological size, the string coupling can be relatively small, and the Euler number $|\chi| \simeq g_s^2 l_P \sim g_s^2 \times 10^{32}$ must be very large. The hierarchy is obtained mainly thanks to the large value of χ, so that lowering the bound on R_c lowers the value of χ. Our actual knowledge of gravity at very large distances indicates [32] that R_c should be of the order of the Hubble radius $R_c \simeq 10^{28}$ cm, which implies $g_s \geqslant 10^{-4}$

and $|\chi| \gtrsim 10^{24}$. A large Euler number implies only a large number of closed string massless particles with no *a priori* constraint on the observable gauge and matter sectors, which can be introduced for instance on D3-branes placed at the position where gravity localization occurs. All these particles are localized at the orbifold fixed points (or where the Euler number is concentrated in the general case), and should have sufficiently suppressed gravitational-type couplings, so that their presence with such a huge multiplicity does not contradict observations. Note that these results depend crucially on the scaling of the width w in terms of the Planck length: $w \sim l_P^{\nu}$, implying $R_c \sim 1/l_P^{2\nu+1}$ in string units. If there are models with $\nu > 1$, the required value of χ will be much lower, becoming $\mathcal{O}(1)$ for $\nu \geqslant 3/2$. In this case, the hierarchy could be determined by tuning the string coupling to infinitesimal values, $g_s \sim 10^{-16}$.

The explicit string realization of localized induced gravity models offers a consistent framework that allows us to address a certain number of interesting physics problems. In particular, the effective UV cutoff and the study of the gravity force among matter sources localized on D-branes. It would be also interesting to perform explicit model building and study in detail the phenomenological consequences of these models and compare to other realizations of TeV strings with compact dimensions.

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